HIghly advanced Probabilistic design and Enhanced Reliability methods for high-value, cost-efficient offshore WIND

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Contents

List of Abbreviations

LCOE	Levelized Cost Of Energy
O&M	Operations and Maintenance
WP	Work Package
T	Task
MILP	Mixed Integer Linear Problem
MIQP	Mixed Integer Quadratic Problem

1 Executive Summary

The HIPERWIND project aims at achieving a reduction in the Levelized Cost of Energy of offshore wind farms, through advancements of basic wind energy science which will lead to reductions in risk and uncertainty. The outcome is cost efficient offshore wind through a reduction in unnecessary use of materials, less unscheduled maintenance, and optimized operating strategy tailored at delivering power with high market value.

To fullfill this objective, among many other activities, we have developped a risk-based Operation and Maintenance model, which uses the improved component reliability modelling established in WP5, to devise Operation and Maintenance strategies which minimize financial risk. We will further use this model to assess the value of wind farm assets depending on their Operation and Maintenance strategy and the state of the electricity market. The models outputs will also be used as an input to the final impact assessment wtudy, which will be carried out to quantitatively verify how the technological achievements of HIPERWIND transform into reduction of LCOE.

The objective of the current deliverable is then to describe the model which has been implemented.

The O&M model can be used to produce the following outputs:

- Compute a predictive optimised long-term schedule of future maintenance operations, on the whole life duration of a wind farm
- Evaluate the loss of energy linked to the maintenances
- Evaluate the cost (and loss of revenue) of a maintenance schedule
- Optimize for a given year and a given number of maintenance operations, the maintenance schedule

In practice we have implemented 2 models:

- The short term model optimises the schedule of maintenances over the year, for a given number of operations to be planned, on a selected price and meteorological conditions scenario. Its outputs are both the dates when to schedule each operation and the cost.
- The long term model optimises the long term schedule, ie. the number of maintenances to schedule every year. It needs as inputs the results of the short term model on every year, every possible combination of number of replacements, averaged on the price and meteorological scenarios. Its outputs are both the replacement schedule but also the optimal cost and revenues.

The purpose of this deliverable is to describe the 2 models, write the mathematical formulations and detail how these formulations are modified in order to obtain problems that can be solved by well-known optimization algorithms. Those algorithms and how they were implemented are described.

The models will be used for the benefits of T6.2 and T6.3., in order to compute the expected avoided loss of electricity that is allowed by better optimising the maintenance schedules (which will be used in the LCOE calculations in T6.3) as well as the market value that optimising the maintenance operations could bring.

2 Introduction

The HIPERWIND project aims at achieving a reduction in the Levelized Cost of Energy of offshore wind farms, through advancements of basic wind energy science which will lead to reductions in risk and uncertainty. The outcome is cost efficient offshore wind through a reduction in unnecessary use of materials, less unscheduled maintenance, and optimized operating strategy tailored at delivering power with high market value.

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In practice we have implemented 2 models, as shown on the figure 2.1

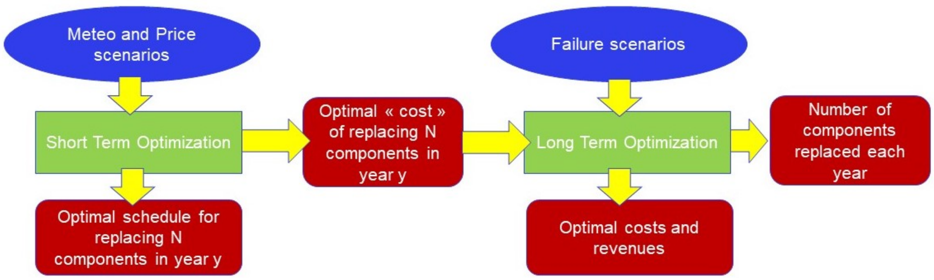

Figure 2.1: HiperWind Optimisation of maintenances models

- The short term model optimises the schedule of maintenances over the year, for a given number of operations to be planned, on a selected price and meteorological conditions scenarios. Its outputs are both the dates when to schedule each operation and the cost.
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but also the optimal cost and revenues.

In section 3.1 we describe the assumptions that were made (in particular linked to simplifications or lack of data). Section 3.2 lists the necessary input data. Then we describe the two optimisation problems in section 3.3. Sections 4 and 5 are dedicated to the mathematical formulation of each problem, as well as the reformulations that were made to reach a formulation which can be solved either by a commercial solver or by an algorithm implemented in the scope of the project.

3 The maintenance scheduling model: data, assumption and functionalities

3.1 Assumptions

- Only major maintenances, ie. replacement of 'big' components (eg. Main Bearings, Gearboxes) are included in the optimization. To avoid the 'curse of dimensionality, it is recommended to keep the number of component categories limited (eg. 2 or 3);
- Components could be replaced by new components or by refurbished ones, at a lower cost. The choice of refurbished or new component is not included in the model. (could be in a further version)
- Even though some planned maintenance may exist, we assume that the operation we are scheduling are always prioritized.
- No ageing factor is applied to the generation of turbines.
- All turbines in a farm are identical, and the conditions applied to all turbines are also identical. This means that, Without maintenance, all turbines of the same farm have the same power output, only depending on meteorological conditions, assuming that these conditions are identical for all turbines in the farm. This means that with one turbine unavailable at hour h, the output will be $P_h = (N_{turbines} - 1) * P_h^{max}$ with $N_{turbines}$ the number of turbines in the farm and P_h^{max} the power depending on meteorological condition of one available turbine.
- Only one category of vessel (Jackup for Teeside) will be considered.
- The time for the vessel to reach the farm from the harbour is neglected.
- Components are always replaced before they fail. This assumption comes from the fact that the expected failure scenarios have a yearly granularity which means we have no information on the period of the year when failures may occur.

3.2 Inputs

In this section, we describe the needed inputs to the model.

- Description of a Maintenance operation for each component category: A maintenance operation is composed of a sequence of 'sub-operations'
  - Each sub-operation has a duration (expressed in days).
  - Sub-operations cannot be interrupted. Between sub-operations, interruptions may occur (due to weather).

  - Each sub-operation is associated to given necessary conditions for accessibility (maximum waves height and maximum wind speed). The sub-operation can occur only when accessibility conditions are fullfilled.
- Constraints
  - Maximum number of operations for each component category in a year
  - Maximum number of parallel operations (=number of vessels) , in total and per component category
  - Maximum duration of a whole maintenance operation for each category of components
- Failure probabilities: Scenarios with the expected number of failures per year
- Maintenance costs (per category of component):
  - Fixed cost depending on the number of components to be replaced during the year
  - The vessel part of the fixed cost is scenarised with scenarios being linked to the delay when the boat is available (if the boat is delayed the cost is lower). Missing details for generating these scenarios.
  - Per day maintenance cost (eg. for team cost, vessel daily cost.....)
  - Waiting cost per day when the vessel is mobilized but nothing is happening due to bad weather conditions
  - Vessel mobilization cost for a maximum period and a maximum number of maintenance operations. This cost can be scenarized to deal with the fact that if the vessel is late, there may be a discount.
- Budget constraints: budget limit for the maintenance expenses on the whole period and each year
- Climate scenarios: hourly correlated time series : wind speed, wave height, max potential generation
- Unavailability schedules (eg. linked to availability of vessels)
- Availability of spare parts: when in the year are each spare parts available
- Any other Not-Before or Not-after constraints

3.3 The Maintenance scheduling Optimisation problem

The maintenance scheduling problem is formulated as a 2 stages optimisation problem as follows:

- Short term:
  - Time horizon: 1 year
  - Time granularity: 1 day
  - Decision variable: When in the year to plan each maintenance operation
  - Objective function: minimize cost / maximize value (cost of maintenance, value of selling electricity on the market)

- Constraints: operation constraints (linked to the availability of materials, boats, how many items can fit on a boat, ...)
- Uncertainties: climate scenarios; price scenarios ; delays when vessels are available
- Results:
  * The cost matrix $C(y, \{N^c_{maint}, c \in C\}, s)$ giving the optimal cost for each year y and each scenario s of replacing N_{maint} components. ($\{N^c_{maint}, c \in C\}$ is the list of number of components of each category to be replaced, C being the list of components (eg MaiBearing, gearbox....)
  * For each year y and each scenario s, the best periods for replacing components, in the case of N_{maint} components
- Solving process: A monte-carlo optimisation will be conducted on all scenarios, for all possible number of maintenance operations to be run (from 0 to $N^{max}_{fail}(y, c)$). For each year/scenario/number of components a mixed integer linear programming will be implemented (or a mixed integer quadratic programming if necessary)

The short term problem will be solved for each year and each possible number of maintenance operations, thus allowing to compute the set of optimal costs of scheduling all possible combinations of number of maintenance operations in all possible years. These costs will be used as inputs to the long-term problem which will compute the optimal number of maintenance operations to be scheduled each year. The short term model is also used to define the optimal schedule in a given year, given the number of operations to plan.

- Long term:
  - Time horizon: N years, where N is the number of remaining year before the expected end of life of turbines. This horizon needs to be consistent with the failure model data. It is be a parameter of the model.
  - Time granularity: 1 year (failure data are given with year granularity)
  - Decision variable: How many items of each category are replaced each year of the horizon
  - Objective function: minimize cost (including maintenance cost, waiting cost, and electricity not sold). The objective function is a function of the costs computed by the short-term model.
  - Constraints:
    * Maximum number of maintenance operations per year
    * Components are always replaced before failure; Here it means that if the failure model predicts a failure of a component in year y, it has to be replaced latest year y-1.
    * Budget constraints ;
    * All constraints in the long-term problem can only be expressed as bounds on the number of operations per category, and bounds on the costs, both for the whole period of per year.

– uncertainties: failures probabilities, which are interpreted as the number of equipment which must have been replaced before year Y;
– Solving process: The long term problem will be solved by a stochastic optimisation approach: deterministic or stochastic dynamic programming.

4 The short term optimisation problem

In the short term, we assume that we know the number of maintenances per component category to be scheduled. The objective is to plan them, while minimising cost and maximising benefits, accounting for uncertainties (climate conditions, electricity prices, delays for vessels availability), and fullfiling operations constraints. The results of this optimisation will be:

- the optimal cost $C^{short}(y, \{N^{maint}_{y,c}, c \in [0, C-1]\})$ for scheduling $\{N^{maint}_{y,c}, c \in [0, C-1]\}$ maintenance operations (where $N^{maint}_{y,c}$ is the number of maintenance operation for components of category c to be scheduled in the year y;
- the optimal schedule $\{t^c_{i,j}, i \in [0, I^c_y - 1], j \in [0, J^c - 1]\}$ of these maintenance operations.

4.0.1 Mathematical formulation of the problem

$$
\begin{aligned}
C^{short}(y, \{N^{maint}_{y,c}, c \in [0, C-1]\}) = \\
\min_{\{t^c_{i,j}, i \in [0, I^c_y-1], j \in [0, J^c-1]\}} \Bigg[FC(\{N^{maint}_{y,c}, c \in [0, C-1]\}) \\
+ WC(\{t^c_{i,j}, i \in [0, I^c_y - 1], j \in [0, J^c - 1]\}) \\
+ LR(\{t^c_{i,j}, i \in [0, I^c_y - 1], j \in [0, J^c - 1]\}) + VM(\{t^c_{i,j}, i \in [0, I^c_y - 1], j \in [0, J^c - 1]\}) \Bigg]
\end{aligned}
$$

with:

- $FC(\{N^{maint}_{y,c}, c \in [0, C-1]\})$ is the fixed part of the maintenance cost, depending only on the number of maintenances. This cost is then fixed in our optimisation problem;
- $WC(\{t^c_{i,j}, i \in [0, I^c_y - 1], j \in [0, J^c - 1]\})$ is the waiting cost, depending on the schedule of the maintenances;
- $LR(\{t^c_{i,j}, i \in [0, I^c_y - 1], j \in [0, J^c - 1]\})$ is the lost revenue, corresponding to periods when some turbines are under maintenance, and then do not produce electricity. This value depends on the schedule of the maintenances;
- $VM(\{t^c_{i,j}, i \in [0, I^c_y - 1], j \in [0, J^c - 1]\})$ is the mobilisation cost; It depends on the number of necessary mobilisation, given that a mobilisation has a fixed cost for a given duration, and can include up to e.g. 5 operations.

The decisions variables are all the dates when a sub-block of a maintenance operation is starting: $\{t^c_{i,j}, i \in [0, I^c_y - 1], j \in [0, J^c - 1]\}$, with $t^c_{i,j}$ being the first day of the jth subblock of the ith maintenance operation for a component of category c.

The optimisation problem can then be simplified to:

$$\min_{\{t^c_{i,j}, i\in[0,I^c_y-1], j\in[0,J^c-1]\}} \Big[WC(\{t^c_{i,j}, i \in [0, I^c_y - 1], j \in [0, J^c - 1]\}) + LR(\{t^c_{i,j}, i \in [0, I^c_y - 1], j \in [0, J^c - 1]\}) + VM(\{t^c_{i,j}, i \in [0, I^c_y - 1], j \in [0, J^c - 1]\}) \Big]$$

which can be written:

$$\min_{\{t^c_{i,j}, i\in[0,I^c_y-1], j\in[0,J^c-1]\}} \Bigg[\sum_{t\in T, c\in C} \Big(w^c(t) * W_{y,c,t} - \sum_{h\in[24*t;24*(t+1)[} [\lambda_{h,s} * \overline{P}_{h,s} * (N_{turbines} - op^c(t)] \Big) + m^c * M_c \Bigg] \tag{R}$$

The constraints of the optimisation problem being:

(C-R-1) All the maintenance sub-blocks are scheduled in the year for each component category:

$$\forall c \in [0; C-1], \forall i \in [0, I^c_y - 1], \forall j \in [0, J^c - 1], t^c_{i,j} \le T_y - 1$$

(C-R-2) Sub-blocks in a maintenance operations are scheduled one after the other:

$$\forall c \in [0; C-1], \forall i \in [0, I^c_y - 1], \forall j \in [0, J^c - 2], t^c_{i,j+1} \ge t^c_{i,j} + d^c_j$$

(C-R-3) Maximum duration of a maintenance operation:

$$\forall c \in [0; C-1], \forall i \in [0, I^c_y - 1], t^c_{i,J^c-1} + d^c_{J^c-1} - t^c_{i,0} \le \overline{D}^c$$

(C-R-4) A turbine is not producing during a maintenance; We will denote $op^c(t)$ the number of maintenance operations which are ongoing at time t with

$$op^c(t) = \sum_{i=0}^{I^c_y-1} op^c_i(t)$$

$op^c_i(t)$ is then defined as follows:

- Before the first day of the first sub-block:

$$\forall c \in [0; C-1], \forall i \in [0, I^c_y - 1], \forall t < t^c_{i,0}, op^c_i(t) = 0$$

- Between the first day of the first sub-block and the last day of the last sub-block of a maintenance operation:

$$\forall c \in [0; C-1], \forall i \in [0, I^c_y - 1], \forall t \in [t^c_{i,0}; t^c_{i,J^c-1} + d^c_{J^c-1} - 1], op^c_i(t) = 1$$

- Between 2 maintenance operations, ie between the last day of the last sub-block of a maintenance and the first day of the first sub-block of the next maintenance operation:

$$\forall c \in [0; C-1], \forall i \in [0, I_y^c - 2], \forall t \in]t_{i,J^c-1}^c + d_{J^c-1}^c - 1; t_{i+1,0}^c[, op_i^c(t) = 0$$

- After the last day of the last sub-block :

$$\forall c \in [0; C-1], \forall i \in [0, I_y^c - 1], \forall t \in]t_{I_y^c-1,J^c-1}^c + d_{J^c-1}^c - 1; T_y[, op_i^c(t) = 0$$

(C-R-5) Maximum number of parallel operations:

$$\forall c \in [0, C-1], \forall t \in [0, T_y - 1], op^c(t) \leq \overline{P^c}$$

$$\forall t \in [0, T_y - 1], \sum_{c \in [0, C-1]} op^c(t) \leq \overline{P}$$

(C-R-6) Waiting periods occur between subblocks; We will denote $w^c(t)$ the number of maintenance operations that are 'waiting' at time t with

$$w^c(t) = \sum_{i=0}^{I_y^c-1} w_i^c(t)$$

$w_i^c(t)$ is then defined as follows:

- Before the last day of the first sub-block: no waiting:

$$\forall c \in [0; C-1], \forall i \in [0, I_y^c - 1], \forall t < t_{i,0}^c + d_0^c - 1, w_i^c(t) = 0$$

- During a sub-block of a maintenance operation: no waiting

$$\forall c \in [0; C-1], \forall i \in [0, I_y^c-1], \forall j \in [0, J^c-1], \forall t \in [t_{i,j}^c; t_{i,j}^c + d_j^c - 1], w_i^c(t) = 0$$

- Between sub-blocks of a maintenance operation: waiting

$$\forall c \in [0; C-1], \forall i \in [0, I_y^c-1], \forall j \in [0, J^c-2], \forall t \in]t_{i,j}^c + d_j^c - 1; t_{i,j+1}^c[, w_i^c(t) = 1$$

- Between maintenance operations: no waiting

$$\forall c \in [0; C-1], \forall i \in [0, I_y^c - 2], \forall t \in]t_{i,J^c-1}^c + d_{J^c-1}^c - 1; t_{i+1,0}^c[, w_i^c(t) = 0$$

- After the last day of the last sub-block of the last maintenance operation: no waiting

$$\forall c \in [0; C-1], \forall i \in [0, I_y^c - 1], \forall t \in]t_{I-1,J^c-1}^c + d_{J^c-1}^c - 1; T_y[, w_i^c(t) = 0$$

(C-R-7) Maximum number of parallel operations:

$$\forall c \in [0, C-1], \forall t \in [0, T_y - 1], w^c(t) \leq \overline{P^c}$$

$$\forall t \in [0, T_y - 1], \sum_{c \in [0, C-1]} w^c(t) \leq \overline{P}$$

(C-R-8) Accessibility conditions: the jth sub-block of a maintenance operation for a component of category c can occur only if the required accessibility condition is fulfilled

$$\forall c \in [0; C-1], \forall i \in [0, I_y^c - 1], \forall t \in [t_{i,0}^c; t_{i,J^c-1}^c + d_{J^c-1}^c - 1] A_{t,s} \geq \underline{A}_j^c$$

(C-R-9) Schedule and availability conditions: the ith maintenance operation cannot start before a specific date,

$$\forall c \in [0; C-1], \forall i \in [0, I_y^c - 1], t_{i,0}^c \geq \underline{T}_i^c$$

(C-R-10) Schedule and availability conditions: the ith maintenance operation cannot end after another specific date,

$$\forall c \in [0; C-1], \forall i \in [0, I_y^c - 1], t_{i,J^c-1}^c + d_{J^c-1}^c - 1 \leq \overline{T}_i^c$$

(C-R-11) Mobilisation cost: a vessel is mobilised for a maximum duration. If it is necessary to mobilise it many times then the cost increases.

$$\forall c \in [0; C-1], m^c = \sum_{i=1}^{I_y^c - 1} z_i^c$$

where:

- if the next operation ends before the end of the mobilisation period, no addtional cost:
$$\text{if } t_{i,J^c-1}^c \leq t_{i-1,0}^c + \overline{DM}^c \text{ then } z_i^c = 0$$
- if the next operation ends after the end of the mobilisation period, there is an addtional cost:
$$\text{if } t_{i,J^c-1}^c > t_{i-1,0}^c + \overline{DM}^c \text{ then } z_i^c = 1$$

where:

- y is the year index (useful for data depending on the year)
- $s \in S$ is the scenario index
- T_y is the number of days of the considered year
- $\{h_t\}$ are the indexes of the hours of the day t, with $\{h_t\} \in [24 * t; 24 * (t+1)[$
- H_y is the number of hours in the year y
- c is the index of the component category (eg. gearbox)
- C is the number of components categories
- $\{N_{y,c}^{maint}, c \in [0, C-1]\}$ is the number of maintenances for each different component category to be scheduled the year y
- $t_{i,j}^c$ is the start date of the jth block of the ith maintenance operation for the component category c
- I_y^c is the number of maintenance operations to be scheduled the year y for the component category c
- J^c is the number of sub-blocks composing a maintenance operation for the component category c
- d_j^c is the duration of the jth sub-block of a maintenance operation for a component of category c

- $\overline{D}^c$ is the maximum duration of a maintenance operation for a component of category c
- $N_{turbines}$ is the number of turbines in the farm
- $op^c(t)$ the number of turbines of category c in maintenance at time t
- $op_i^c(t) = 1$ if the ith maintenance operation in the category c is occurring at t, $op_i^c(t) = 0$ if not.
- $w^c(t)$ the number of turbines which are 'in the middle' of a maintenance operation (waiting)
- $w_i^c(t) = 1$ if t corresponds to a waiting period during the ith maintenance operation in the category c, $w_i^c(t) = 0$ if not.
- m^c the number of necessary vessel mobilisations
- M^c the unitary mobilisation cost of the vessel
- $\overline{DM}^c$ the maximum duration of one vessel mobilisation (which can be used for more than one operation)
- $\overline{P}$ the maximum number of parallel operations
- $\overline{P}^c$ the maximum number of parallel operations for components of category c
- $W_{y,c,t}$ is the cost of waiting during 1 day, at day t, in the year y for the component category c
- $\lambda_{h,s}$ is the marginal cost of electricity for the hour h of the year, in the scenario s
- $\overline{P}_{h,s}$ is the power generated by 1 turbine at hour h for the scenario s if the turbine is available.
- $A_{t,s}$ describes the accessibility conditions. $A_{t,s}$ can take a limited number of integer values $\{0, A^i, i \in [0; N_A]\}$ where N_A is the number of possible accessibility situations; The biggest $A_{t,s}$ is, the more accessible the conditions are, starting from $A_{t,s} = 0$ where it is impossible to conduct any kind of operation to $A_{t,s} = \max_{A^i, i \in [0;N_A]}$ where all operations are possible.
- $\underline{A}_j^c$ is the minimum required accessibility conditions for the jth sub-block of an operation for a component of category c. $\underline{A}_j^c \in \{0, A^i, i \in [0; N_A]\}$

4.0.2 Re-Formulation of the problem as a MILP or a MIQP

We define the following variables as being the decision variables of the optimisation problem:

- $s_{c,t,i,j}$ is a binary variable which is equal to 1 if the jth sub-block of the ith maintenance operation for the component of category c starts at time t.
- $z_{c,i}$ is a binary variable which is equal to 1 if ith maintenance operation ends before the mobilisation period corresponding to the (i-1)th operation. $z_{c,0} = 0$. Including those variables will lead to transforming the linear problem into a quadratic problem.

As we are optimising over one year, with a daily granularity for the decision variables, assuming that there may be a maximum of 5 different categories of components, that we cannot schedule more than 10 maintenance operations for each component category and that a maintenance operation will be composed of max 5 subblocks, the number of binary variables will never exceed $365 * 5 * 10 * 5 < 100000$.

We can then compute op_t^c, w_t^c and m^cas follows:

$$op^c(t) = \sum_{i=0}^{I_y^c - 1} \left[\sum_{\theta=0}^{t} s_{c,\theta,i,0} - \sum_{\theta=0}^{t-d_{J^c-1}} s_{c,\theta,i,J^c-1} \right] \tag{4.1}$$

$$w^c(t) = \sum_{i=0}^{I_y^c-1}\left[\sum_{\theta=0}^{t} s_{c,\theta,i,0} - \sum_{\theta=0}^{t-d_{J^c-1}} s_{c,\theta,i,J^c-1} - \sum_{j=0}^{J^c-1}\left(s_{c,t,i,j} + \sum_{\theta=t-d_j^c}^{t-1} s_{c,\theta,i,j}\right)\right]$$

simplified as:

$$w^c(t) = \sum_{i=0}^{I_y^c-1}\left[\sum_{\theta=0}^{t} s_{c,\theta,i,0} - \sum_{\theta=0}^{t-d_{J^c-1}} s_{c,\theta,i,J^c-1} - \sum_{j=0}^{J^c-1}\sum_{\theta=t-d_j^c+1}^{t} s_{c,\theta,i,j}\right] \tag{4.2}$$

$$m^c = \sum_{i=1}^{I_y^c-1}[1-z_i^c] = I_y^c - \sum_{i=1}^{I_y^c-1} z_i^c \tag{4.3}$$

which allows to write the objective as a function F of the binary variables $s^{i,j,t,c}$:

$$\min_{\{s_{c,t,i,j}, i<I_y, j<J^c, t<T_y, c<C\}}\left[\sum_{t\in T, c\in C}\left(w^c(t)*W_{y,c,t} - \sum_{h\in[24*t;24*(t+1)[}[\lambda_{h,s}*\overline{P}_{h,s}*(N_{turbines}-op^c(t)]\right) + \sum_{c\in C} m^c * M_c\right] \tag{4.4}$$

with F the function to minimise and

$$F = \sum_{c=0}^{C-1} F_c$$

We will denote in the following:

$$V_{y,t,s} = \sum_{h\in[24*t;24*(t+1)[}\left[\lambda_{h,s}*\overline{P}_{h,s}\right]$$

We can the write

$$F_c = m^c * M_c + \sum_{t=0}^{T_y-1}[w^c(t)*W_{y,c,t} + op^c(t)*V_{y,t,s}] - \sum_{t=0}^{T_y-1}[N_{turbines}*V_{y,t,s}] \tag{4.5}$$

where

$$\sum_{t=0}^{T_y-1}[N_{turbines}*V_{y,t,s}]$$

is a fixed term, denoted F_c^{fixed}

We then have $F_c = F_c^v - FC_c^{fixed}$

F_c^v can be written:

$$F_c^v = \sum_{i=0}^{I_y^c-1} \left[\sum_{t=0}^{T_y-1} \left((W_{y,c,t} + V_{y,t,s}) \sum_{\theta=0}^{t} s_{c,\theta,i,0} - (W_{y,c,t} + V_{y,t,s}) \sum_{\theta=0}^{t-d_{J^c-1}} s_{c,\theta,i,J^c-1} - W_{y,c,t} \sum_{j=0}^{J^c-1} \sum_{\theta=t-d_j^c+1}^{t} s_{c,\theta,i,j} \right) - M_c z_{c,i} \right] \tag{4.6}$$

We can now reorder F_c^v as follows:

$$F_c^v = \sum_{i=0}^{I_y^c-1} \left\{ \sum_{t=0}^{T_y-1} \left[\left(\sum_{\theta=t}^{T_y-1} (W_{y,c,\theta} + V_{y,\theta,s}) \right) s_{c,t,i,0} - \left(\sum_{\theta=t+d_{J^c-1}}^{T_y-1} (W_{y,c,\theta} + V_{y,\theta,s}) \right) s_{c,t,i,J^c-1} - \sum_{j=0}^{J^c-1} \left(\sum_{\theta=t}^{t+d_{J^c-1}-1} W_{y,c,\theta} \right) s_{c,t,i,j} \right] - M_c z_{c,i} \right\} \tag{4.7}$$

Note that we can write $t_{i,j}^c = \sum_{t=0}^{T-1} t * s_{i,j,t,c}$

Finally we need to reformulate the accessibility conditions: $A_{s,t,c,j}$ is a binary data of which value is 1 when for the scenario s at timestep t (expressed in days) it is possible to operate the jth sub-block of a maintenance operation for a component of category c, and 0 if not. This is equivalent to when $A_{t,s} \geq \underline{A}_j^c$.

The constraints of the problem (R) can be written as follows

(C-M-1) all subblocks of maintenance operations are scheduled once It corresponds to constraint (C-R-1)

$$\forall c \in [0; C-1], \forall i \in [0, I_y^c - 1], \forall j \in [0, J^c - 1], \sum_{t=0}^{T_y-1} s_{c,t,i,j} = 1$$

(C-M-2) all maintenance operations are scheduled.

$$\forall c \in [0; C-1], \forall j \in [0, J^c - 1], \sum_{t=0}^{T_y-1} \sum_{i=0}^{I_y^c-1} s_{c,t,i,j} = N_{y,c}^{maint}$$

(C-M-3) all subblocks for each maintenance operations are scheduled.

$$\forall c \in [0; C-1], \forall i \in [0, I_y^c - 1], \forall j \in [0, J^c - 1], \sum_{t=0}^{T_y-1} s_{c,t,i,j} = 1$$

This constraint is useless as already included in (C-M-1)

(C-M-4) for each maintenance, sub-blocks are scheduled in the required order. This is equivalent to constraint (C-R-2)

$$\forall c \in [0; C-1], \forall i \in [0, I_y^c - 1], \forall j \in [0, J^c - 2], \sum_{t=0}^{T-1} t * s_{c,t,i,j} - \sum_{t=0}^{T-1} t * s_{c,t,i,j+1} \leq 1 - d_j^c$$

(C-M-5) maximum duration of a maintenance operation. It corresponds to constraint (C-R-3)

$$\forall c \in [0; C-1], \forall i \in [0, I_y^c - 1], \sum_{t=0}^{T-1} (t * s_{c,t,i,J^c-1} - t * s_{c,t,i,0}) \leq \overline{D}^c - d_{J^c-1}^c$$

(C-M-6) is related to the maximum number of parallel operations (for all kinds of maintenances). It corresponds to constraint (C-R-5)

$$\forall t \in [0, T_y - 1], \sum_{c=0}^{C-1} \sum_{i=0}^{I_y^c-1} \left[\sum_{\theta=0}^{t} s_{c,\theta,i,0} - \sum_{\theta=0}^{t-d_{J^c-1}} s_{c,\theta,i,J^c-1} \right] \leq \overline{P}$$

This can be also be written with a simpler constraint:

$$\forall t \in [0, T_y - 1], \sum_{c=0}^{C-1} \sum_{i=0}^{I_y^c-1} \sum_{j=0}^{J^c-1} \sum_{\theta=t+1-d_{J^c-1}}^{t} s_{c,\theta,i,j} \leq \overline{P}^c$$

(C-M-7) is related to the maximum number of parallel operations for maintenances for components of category c. It corresponds to constraint refMaxParallel2

$$\forall t \in [0, T_y - 1], \forall c \in [0, C-1], \sum_{i=0}^{I_y^c-1} \left[\sum_{\theta=0}^{t} s_{c,\theta,i,0} - \sum_{\theta=0}^{t-d_{J^c-1}} s_{c,\theta,i,J^c-1} \right] \leq \overline{P}^c$$

This can be also be written with a simpler constraint:

$$\forall t \in [0, T_y - 1], \forall c \in [0, C-1], \sum_{i=0}^{I_y^c-1} \sum_{j=0}^{J^c-1} \sum_{\theta=t+1-d_{J^c-1}}^{t} s_{c,\theta,i,j} \leq \overline{P}^c$$

(C-M-8) is related to the accessibility conditions. It corresponds to constraint (C-R-8).

$$\forall t \in [0, T_y-1], \forall c \in [0; C-1], \forall j \in [0, J^c-1], \sum_{i=0}^{I_y^c-1} \left[s_{c,t,i,j} + \sum_{\theta=t-d_j^c+1}^{t-1} s_{c,\theta,i,j} \right] \leq A_{s,t,c,j}$$

(C-M-9) is related to the scheduling conditions (not after). It corresponds to constraints (C-R-10)

$$\forall c \in [0; C-1], \forall i \in [0, I_y^c - 1], \sum_{t=0}^{T_y-1} (-1) * t * s_{c,t,i,0} \leq (-1) * \underline{T}_i^c$$

(C-M-10) is related to the scheduling conditions (not before). It corresponds to constraints (C-R-9)

$$\forall c \in [0; C-1], \forall i \in [0, I_y^c - 1], \sum_{t=0}^{T-1} t * s_{c,t,i,J^c-1} \leq \overline{T}_i^c - d_{J^c-1}^c$$

(C-M-11) is related to the number of necessary vessel mobilization. It corresponds to constraints (C-R-11). This constraint is quadratic.

$$\forall c \in [0; C-1], \forall i \in [1; I_y^c - 1], \sum_{t=0}^{T-1} (t * s_{c,t,i,J^c-1} - t * s_{c,t,i-1,0}) * z_{c,i} \leq \overline{DM}^c$$

(C-M-12) The constraints (C-R-4) and (C-R-6) or the problem (R) are embedded in the formulation of the problem (M)

4.0.3 The short term problem expressed as a MILP or a MIQP with binary variables

The problem to be solved is then:

$$\begin{aligned}
&C^{short}(y, \{N^{maint}_{y,c}, c \in [0, C-1]\}) = \\
&\min_{\{s_{c,t,i,j}\}} \sum_{c=0}^{C-1} \sum_{i=0}^{I^c_y-1} \{ \sum_{t=0}^{T_y-1} \left[\left(\sum_{\theta=t}^{T_y-1} (W_{y,c,\theta} + V_{y,\theta,s}) \right) s_{c,t,i,0} \right. \\
&\left. - \left(\sum_{\theta=t+d_{J^c-1}}^{T_y-1} (W_{y,c,\theta} + V_{y,\theta,s}) \right) s_{c,t,i,J^c-1} - \sum_{j=0}^{J^c-1} \left(\sum_{\theta=t}^{t+d_{J^c-1}-1} W_{y,c,\theta} \right) s_{c,t,i,j} \right] - M_c z_{c,i} \} \\
&- \sum_{c=0}^{C-1} \sum_{t=0}^{T_y-1} (N_{turbines} * V_{y,t,s}) + I^c_y * M^c
\end{aligned} \tag{M}$$

with the following constraints:

(C-M-1) $\forall c \in [0; C-1], \forall i \in [0, I^c_y - 1], \forall j \in [0, J^c - 1],$

$$\sum_{t=0}^{T_y-1} s_{c,t,i,j} = 1$$

(C-M-2) $\forall c \in [0; C-1], \forall j \in [0, J^c - 1],$

$$\sum_{t=0}^{T_y-1} \sum_{i=0}^{I^c_y-1} s_{c,t,i,j} = N^{maint}_{y,c}$$

(C-M-3) $\forall c \in [0; C-1], \forall i \in [0, I^c_y - 1],$

$$\sum_{t=0}^{T_y-1} \sum_{j=0}^{J^c-1} s_{c,t,i,j} = J^c$$

(C-M-4) $\forall c \in [0; C-1], \forall i \in [0, I^c_y - 1], \forall j \in [0, J^c - 2],$

$$\sum_{t=0}^{T-1} t * s_{c,t,i,j} - \sum_{t=0}^{T-1} t * s_{c,t,i,j+1} \leq 1 - d^c_j$$

(C-M-5) $\forall c \in [0; C-1], \forall i \in [0, I^c_y - 1],$

$$\sum_{t=0}^{T-1} (t * s_{c,t,i,J^c-1} - t * s_{c,t,i,0}) \leq \overline{D}^c - d^c_{J^c-1}$$

(C-M-6) $\forall t \in [0, T_y - 1],$

$$\sum_{c=0}^{C-1} \sum_{i=0}^{I^c_y-1} \left[\sum_{\theta=0}^{t} s_{c,\theta,i,0} - \sum_{\theta=0}^{t-d_{J^c-1}} s_{c,\theta,i,J^c-1} \right] \leq \overline{P}$$

(C-M-7) $\forall t \in [0, T_y - 1], \forall c \in [0, C - 1],$

$$\sum_{i=0}^{I_y^c-1} \left[\sum_{\theta=0}^{t} s_{c,\theta,i,0} - \sum_{\theta=0}^{t-d_{J^c-1}} s_{c,\theta,i,J^c-1} \right] \leq \overline{P}^c$$

(C-M-8) $\forall t \in [0, T_y - 1], \forall c \in [0; C - 1], \forall j \in [0, J^c - 1],$

$$\sum_{i=0}^{I_y^c-1} \left[s_{c,t,i,j} + \sum_{\theta=t-d_j^c+1}^{t-1} s_{c,\theta,i,j} \right] \leq A_{s,t,c,j}$$

(C-M-9) $\forall c \in [0; C - 1], \forall i \in [0, I_y^c - 1],$

$$\sum_{t=0}^{T_y-1} (-1) * t * s_{c,t,i,0} \leq (-1) * \underline{T}_i^c$$

(C-M-10) $\forall c \in [0; C - 1], \forall i \in [0, I_y^c - 1],$

$$\sum_{t=0}^{T-1} t * s_{c,t,i,J^c-1} \leq \overline{T}_i^c - d_{J^c-1}^c$$

(C-M-11) $\forall c \in [0; C - 1], \forall i \in [1; I_y^c - 1],$

$$\sum_{t=0}^{T-1} (t * s_{c,t,i,J^c-1} - t * s_{c,t,i-1,0}) * z_{c,i} \leq \overline{DM}^c$$

where:

- The constraint (C-M-1) means that all subblocks of maintenance operations are scheduled once
- The constraint (C-M-2) means that all maintenance operations are scheduled.
- The constraint (C-M-3) means that all subblocks for each maintenance operations are scheduled. It corresponds to constraint (C-R-1)
- The constraint (C-M-4) means that for each maintenance, sub-blocks are scheduled in the required order. This is equivalent to constraint (C-R-2)
- The constraint (C-M-5) is related to the maximum duration of a maintenance operation. It corresponds to constraint (C-R-3)
- The constraint (C-M-6) and (C-M-7) are related to the maximum number of parallel operations (for all kinds of maintenances or for maintenances for components of category c. It corresponds to constraint (C-R-5) and (C-R-7)
- The constraint (C-M-8) is related to the accessibility conditions. It corresponds to constraint (C-R-8). Those constraints can be skipped for subblocks/ timesteps wih accessible conditions.
- The constraints (C-M-9) and (C-M-10) are related to thescheduling conditions (not after/nt before). They correspond to constraints (C-R-10) and (C-R-9)

- The constraints (C-M-11) are related to mobilisation periods. They correspond to constraints (C-R-11)
- The constraints (C-R-4) and (C-R-6) or the problem (R) are embedded in the formulation of the problem (M)

4.0.4 Solving the short term problem with a MILP or MIQC solver

Without the mobilisation constraints, the short term problem is solved using a MILP solver to solve the problem as formulated above. With this constraint it is solved with a MIQC solver.

The objective function (M) can be written:

$$\min_{\{x_{c,t,i,j}\}} \sum_{c=0}^{C-1} \sum_{i=0}^{I_y^c-1} \left[\sum_{j=0}^{J^c-1} \sum_{t=0}^{T_y-1} a_{c,t} x_{c,t,i,j} + b_c z_{c,i} \right] - K \tag{4.8}$$

with:

- for $j = 0$:

$$a_{c,t} = \sum_{\theta=0}^{T_y-1} (W_{y,c,\theta} + V_{y,\theta,s})$$

- for $j = J^c - 1$:
  - for $t \leq T_y - 1 - d_{J^c-1}$:

$$a_{c,t} = (-1) * \sum_{\theta=t+d_{J^c-1}}^{T_y-1} (W_{y,c,\theta} + V_{y,\theta,s})$$

  - for $t > T_y - 1 - d_{J^c-1}$:

$$a_{c,t} = 0$$

- for $0 < j < J^c - 1$:
  - for $t \leq T_y - d_{J^c-1}$:

$$a_{c,t} = (-1) * \sum_{\theta=t}^{t+d_{J^c-1}-1} W_{y,c,\theta}$$

  - for $t > T_y - d_{J^c-1}$:

$$a_{c,t} = (-1) * \sum_{\theta=t}^{T_y-1} W_{y,c,\theta}$$

- for $0 < i < I_y^c$:

$$b_c = (-1) * M^c$$

with contraints (C-M-1) to (C-M-11).

5 The long term optimization problem

In the long term, we compute the number of maintenances per component category and per year, while minimising cost and maximising benefits. We have implemented 2 versions:

- Solve the deterministic problem on each of the uncertainties scenarios, and compute the average;
- Solve the stochastic problem on a scenario tree.

5.1 The long term mathematical model

$$\min_{N_{y,c,s}, y\in[0;Y[, c\in[0,C[, s\in[0,S[} \left(\mathbb{E}_{s\in[0,S[} \left[\sum_{y=0}^{Y-1} \sum_{c=0}^{C-1} C(y, \{N_{y,c,s}, c \in [0, C[\}) \right] \right)$$

$C(y, \{N_{y,c,s}, c \in [0, C[\})$ is the mean optimal cost of scheduling $N_{y,c,s}$ operations for the component category c in the year y.

The decision variables are $\{N_{y,c,s}, y \in [0; Y[, c \in [0, C[, s \in [0, S[\}$, where $N_{y,c,s}$ is the number of maintenance operations scheduled the year y for the component category c in the scenario s. The constraints are the following:

- Enough maintenance operations are scheduled so that failures occur after replacements:
$$\forall y \in [0; Y[, \forall s \in [0, S[, \sum_{x=0}^{y} N_{x,c,s} \geq \sum_{x=0}^{y} \underline{N_{y,s}^{cum}}$$
- Upper bound on the number of replacements each year for each category:
$$\forall y \in [0; Y[, \forall s \in [0, S[, \forall c \in [0; C[, N_{y,c,s} \leq \overline{N_y^c}$$
- Upper bound on the total number of replacements each year:
$$\forall y \in [0; Y[, \forall s \in [0, S[, \sum_{c=0}^{C-1} N_{y,c,s} \leq \overline{N_y}$$
- Upper bound on the budget per year:
$$\forall y \in [0; Y[, \forall s \in [0, S[, \sum_{c=0}^{C-1} C(y, s, N_{y,c,s}) \leq \overline{B_y}$$
- Upper bound on the total budget:
$$\forall s \in [0, S[, \sum_{y=0}^{Y-1} \sum_{c=0}^{C-1} C(y, s, N_{y,c,s}) \leq \overline{B}$$
- Upper bound on the total number of replacements:
$$\forall s \in [0, S[, \sum_{y=0}^{Y-1} \sum_{c=0}^{C-1} N_{y,c,s} \leq \overline{CN}$$

- Upper bound on the total number of replacements of each category:

$$\forall s \in [0, S[, \forall c \in [0, C[, \sum_{y=0}^{Y-1} N_{y,c,s} \leq \overline{CN^c}$$

where:

- Y is the number of years of the horizon;
- C is the number of components categories;
- S is the number of scenarios (note that we are not referring to the same scenarios as in the short term model as here the uncertainties are the failure probabilities;
- $\overline{CN}$ is the maximum possible number of operations to be scheduled
- $\overline{CN^c}$ is the maximum possible number of operations to be scheduled for a component of any category
- $CN^c_{y,s}$ is the cumulative number of expected failures occurred before year y for the component category c in the scenario s;
- $\overline{N^c_y}$ is the maximum number of operations that can be scheduled for the component category c during the year y
- $\overline{N_y}$ is the maximum number of operations that can be scheduled during the year y
- $\overline{B_y}$ is the maximum budget that can be spent during the year y
- $\overline{B}$ is the maximum budget that can be spent during the whole period.
- $C(y, \{N_{y,c,s}, c \in [0, C[\})$ are the costs computed by the short term model.

5.2 Solving the long term problem with dynamic programming

2 options are implemented: solving each scenario via deterministic dynamic programming, or solving the whole problem by stochastic dynamic programming.

5.2.1 Probabilising the scenarios

From the S^c_{tot} failure scenarios of each component, we obtain S^c_{prob} scenarios, where scenario s has probability π_s. We then transform these scenarios which give the number of failures $F^c_{s,y}$ for the component c in year y for scenario s in scenarios of cumulated failures $CF^c_{s,y}$, where

$$\forall s \in [0, S[, \forall c \in [0, C[, CF^c_{s,y} = \sum_{x=0}^{y} F^c_{s,y}$$

5.2.2 Deterministic dynamic programming

The dynamic programming is composed of 2 steps:

- Backward step, where the bellman values for each cumulated number of maintenances at each year is computed, from year $Y-1$ to year 0
- Forward step, where the optimal number of maintenances at each year are computed, from year 0 to year $Y-1$

We will denote x the states of the dynamic programming, x being the set of cumulated maintenances since year 0 for all components, with $x = (x^c, c \in [0, C[)$. We will denote $BV(x, y)$ the Bellman value of state x at year y

1. **Backward step**
   - Compute the ranges of reachable cumulated maintenances for each year: for each component c, the possible number of cumulated maintenances is $[\underline{CM^c_{s,y}}, \overline{CM^c_{s,y}}]$, where:
     - $\underline{CM^c_{s,y}} = CF^c_{y,s}$
     - $\overline{CM^c_{s,y}} = \sum_{t=0}^{y} \overline{N^c_y}$

     $x \in range(s, y)$ means:

     $$\forall c \in [0, C[, x^c \in [\underline{CM^c_{s,y}}, \overline{CM^c_{s,y}}]$$

   - Initialise Bellman Values for the last year:

     $$\forall x \in range(s, Y-1), BV(x, Y-1) = 0$$

   - compute Bellman Value for year y:

     $$\forall y \in [Y, 2, 0], \forall x \in range(y), BV(x, y) = \min_{u \in Tr(x,y)} [C(y+1, u) + BV(x+u, y+1)]$$

     where u is the transition (ie the number of maintenances to schedule for each component category), with $u = (u^c, c \in [0, C[)$, $Tr(x, y)$ is the set of possible transitions (ie possible number of maintenances to schedule in year $y+1$ from state x at year y, ie. given that the cumulated number of transitions at year y is x $u \in Tr(x, y)$ means:

     $$\forall c \in [0, C[, u^c \in [\max(0, \underline{CM^c_{s,y+1}} - x^c), \min(\overline{N^c_{y+1}}, \overline{CM^c_{s,y+1}} - x^c]$$

2. **Forward step**
   - Initialize x at year 0: $x_0 = 0$, meaning $\forall c \in [0, C[, x^c_0 = 0$
   - Compute u and x at year y:

     $$u_{y+1} = Argmin_{u \in Tr(x_y, y+1)} [C(y+1, u) + BV(x_y + u, y+1)]$$

     $$x_{y+1} = x_y + u_{y+1} : \forall c \in [0, C[x^c_{y+1} = x^c_y + u_{y+1}$$

   - The optimal cost of scenario s is then the cost computed at year 0:

     $$Cost = \min_{u \in Tr(x_0, 0)} [C(0, u) + BV(x_0 + u, 0)]$$

5.2.3 Stochastic dynamic programming

The stochastic dynamic programming is computed over a scenario tree. This tree is created out of the probabilised scenarios and has the following characteristics:

- It is composed of N nodes (set Nodes), each one with probability π_n
- There are N_y nodes at year y (the set Nodes(y)) with $\sum_{n \in Nodes(y)} \pi_n = 1$
- there is only one (fictive) node $root$ at year -1
- a node has only one father but can have many sons, with:

$$\forall n \in \cup_{y \in [0, Y-2]}, \sum_{m \in sons(n)} \pi_m = \pi_n$$

$$\forall m \in sons(n), father(m) = n$$

- The node n is constructed out of scenarios which have the same values at the year corresponding to the node n and at all past years. We denote $S(n)$ the set of scenarios belonging to node n

$$\forall n \in Nodes, \forall s \in S(n), \forall c \in [0, C[, F^c_{s,y(n)} = F^c_n$$

- The cumulated number of expected failures before node n (included) is then CF^c_n, while the number of expected failures at node n is F^c_n,

The stochastic dynamic programming is composed of 2 steps:

- Backward step, where the bellman values for each cumulated number of mainteances at each year is computed, from year $Y - 1$ to year 0
- Forward step, where the optimal number of maintenances at each year are computed, from year 0 to year $Y - 1$

We will denote x the states of the dynamic programming, x being the set of cumulated maintenances since year 0 for all components, with $x = (x^c, c \in [0, C[)$. We will denote $BV(x, n)$ the bellman value of state x at node n

1. **Backward step**
   - Compute the ranges of reachable cumulated maintenances for each node: for each component c, the possible number of cumulated failures is $[\underline{CF^c_{s,n}}, \overline{CF^c_{s,n}}]$, where:
     - $\underline{CM^c_n} = CF^c_n$
     - $\overline{CM^c_n} = \sum_{t=0}^{y}(n)\overline{N^c_y}$

     $x \in range(n)$ means:

$$\forall c \in [0, C[, x^c \in [\underline{CM^c_n}, \overline{CM^c_n}]$$

   - Initialise Bellman Values for the last year:

$$\forall n \in Nodes(Y - 1), \forall x \in range(n), BV(x, n) = 0$$

   - compute Bellman Value for node n:

$$\forall y \in [Y, 2, 0], \forall n \in Nodes(y), \forall x \in range(n),$$
$$BV(x, n) = \frac{1}{\pi_n} \sum_{m \in sons(n)} \pi_m \min_{u \in Tr(x,m)} [C(y(m), u) + BV(x + u, y(m))]$$

where u is the transition (ie the number of maintenances to schedule for each component category), with $u = (u^c, c \in [0, C[)$, $Tr(x, m)$ is the set of possible transitions (ie possible number of maintenances to schedule at node $m \in sons(n)$ from state x at node n, ie. given that the cumulated number of transitions at node n is x $u \in Tr(x, m)$ means:

$$\forall c \in [0, C[, u^c \in [\max(0, \underline{CM^c_m} - x^c), \min(\overline{N^c_{y(m)}}, \overline{CM^c_m} - x^c]$$

2. **Forward step**
   - Initialize x at year 0: $x_0 = 0$, meaning $\forall c \in [0, C[, x^c_0 = 0$
   - Compute u and x at year y:

$$\forall y \in [1, Y[, \forall n \in Nodes(n),$$
$$u_n = Argmin_{u \in Tr(x_{father(n)}, n)} \left[C(y, u) + BV(x_{father(n)} + u, n) \right]$$

$$x_n = x_{father(n)} + u_n : \forall c in [0, C[x^c_n = x^c_{father(n)} + u_n$$

   - The optimal cost of scenario s is then the cost computed at year 0:

$$Cost = \min_{u \in Tr(x_0, 0)} [C(0, u) + BV(x_0 + u, 0)]$$

6 Implementation

6.1 An Operation and Maintenance tool

For the benefits of the Hiperwind project, both the short-term and long-term models have been implemented and tested.

- Short term optimization:
  - Implemented in python with an interface through pyomo for the MILP and MIQP which can be solved by either GLPK or CPLEX
  - Tests have been conducted on a Linux machine with 8 CPU and 16Go RAM
  - Solving one instance on one year / one scenario takes approximately 1 second in the MILP case and up to 3 minutes in the MIQP case
  - For a given year and a given number of replacements to schedule for each component, the tool solves the problem for each scenario (price scenario, meteorological scenario, delay scenario) and computes the average cost, as well as the average replacement schedule.
  - The model can also be used on a unique scenario in order to obtain a feasible mmaintenance schedule.
- Long term optimization:
  - A deterministic and a stochastic version were Implemented in python
  - Tests have been conducted on a Linux machine with 8 CPU and 16Go RAM

   – The stochastic optimisation takes as inputs the average optimal costs on all years and all combinations of possible number of operations each year as input and computes the optimal number of maintenance operations per year and component category, given the expected failures probabilities.

The implementation consists of 2 python modules, one dedicated to short-term optimisation and one dedicated to long-term optimisation, a yaml parameters and main data file and a set of csv input and output files. No visual interface is available. The implemented sofytware can be ran on any computer, provided that python3 is installed and that the python packages pandas, numpy, os, yaml, math, datetime, calendar, itertools, sys and pyomo are available and installed.

6.2 Inputs

The inputs include:

- A parameter and main data file, implemented as a yaml file. An example with fake data is given below (figure 6.1). This file contains the non timeseries data, in particular the costs and the constraints.
- A csv file for the meteorological data. This files includes one column per data category: Wind speed at 10 m, wave height, maximum power of a turbine (for the given wind speed), and one row per hour. It contains a number of yearly scenarios: the first 8760 rows are the data from Jan 1st 0h to Dec. 31 23h for the first scenario, followed by the second scenario...
- A csv file containing the failure probability data per component catagory and location. This file includes one columns per scenario and one row per future year. The values are the number of expected failures for the given component, scenario and year.
- A serie of csv files containing the prices of electricity in the region where the farm is located: one file per year, containing 1 row per hour, 1 column with the timestamp (format yyyy:mm:dd HH:MM), 1 column per price scenario with the values in €/MW.

6.3 Running the tool

The short term model can be launched in parallel on a serie of :

- years (namely from 2023 to 2048);
- price scenarios;
- meteorological scenarios;
- number of components to replace

6.4 Outputs

6.4.1 Outputs of the short term model

The short term model runs on one given year, and can optimise either the schedule for replacing a given number of components of each category in one given scenario, or can optimise (sequencially) the schedules for replacing any possible combination of number of

```
# path
path: '/home/XXX/HyperWind/IN/'  # path where the input data are located
outpath: '/home/XXX/HyperWind/OUT/'  # path where the output data are located
LogFile: 'HyperWindLog.txt' # logfile

meteodata: 'WindSpeed_WaveHeight.csv'  # name of the file containing the meteo data
CostFile: 'ShortTermCosts.csv' # Average optimal costs :output of the short-term model
Prices:
    Dir: '/home/XX/HyperWind/Prices/' # path where the prices are located
FirstYear: 2023  # first year of the study
LastYear: 2045  # last year of the study
Categories: ['Gearbox','MainBearing'] # list of components whose maintenance to optimise
MaxOperations: 3 # max number of parallel operations allowed at a given day
NumberScenariosMeteo: 200 # number of meteo scenario to onclude in the simulations
NumberScenariosDelay: 4 # number of delay scenarios to include
NumberScenariosPrice: 50 # number of price scenarios to include
MaxPower: 18 # max power of 1 turbine

PerCategory:  # characteristics of maintenances for each category
    Gearbox:  # data below are fake data, only used as an example
        failuredata: 'FailureProbabilities_Gearbox.csv'
        NbScenarios: 500
        NumberBlocks: 3 # number of blocks of the sequence of one maintenance operation
        Duration: [2,1,5,4] # duration of each blocks. This list must be of dimension NumberBlocks
        MaxDuration: 50  # maximum duration of a maintenance operation, incuding waiting between blocks
        MaxOperations: 2 # maximum parallel operations for the category
        MaxWaveHeight: [1.5,2.1,1.8,1.1]  # max wave height under which the jth block is possible
        MaxWindSpeed10m: [10.0,15.0,12.0,8.0] # max wind speedt under which the jth block is possible
        NotBefore: [0,20,50]  # the ith elementis the day before which operation i is not allowed
        NotAfter: [100,200,300] # the ith element is the day before which operation i has to be finished
        FixedCost: [2000,850,610] # The ith value is the cost of 1 operation if i are scheduled
        PerDayOpCost: 48.0  #per day operation cost
        PerDayWaitCost: 23.0 # per day waiting cost
        MobilisationDuration: 45  # maximum duration of vessel mobilisation
        MobilisationCost: 3500.0  # cost for mobilisation of vessel for the whole duration
    MainBearing: # data below are fake data, only used as an example
        failuredata: 'FailureProbabilities_Gearbox.csv'
        NbScenarios: 500
        NumberBlocks: 3 # number of blocks of the sequence of one maintenance operation
        Duration: [2,1,5,4] # duration of each blocks. This list must be of dimension NumberBlocks
        MaxDuration: 50  # maximum duration of a maintenance operation, incuding waiting between blocks
        MaxOperations: 2 # maximum parallel operations for the category
        MaxWaveHeight: [1.5,2.1,1.8,1.1]  # max wave height under which the jth block is possible
        MaxWindSpeed10m: [10.0,15.0,12.0,8.0] # max wind speedt under which the jth block is possible
        NotBefore: [0,20,50]  # the ith elementis the day before which operation i is not allowed
        NotAfter: [100,200,300] # the ith element is the day before which operation i has to be finished
        FixedCost: [2000,850,610] # The ith value is the cost of 1 operation if i are scheduled
        PerDayOpCost: 48.0  #per day operation cost
        PerDayWaitCost: 23.0 # per day waiting cost
        MobilisationDuration: 45  # maximum duration of vessel mobilisation
        MobilisationCost: 3500.0  # cost for mobilisation of vessel for the whole duration
```

Figure 6.1: Main input data file

components of each category (given a maximum number for each category) for all the scenarios. The results are available in the same output files.

- Optimal schedule: one csv file with one row per sub-block of maintenance operation and 7 columns:
  - Column 0: index of the sub-operation
  - Column 1: number of replacements per component category: (1,4) means that there are 2 component categories (eg. gearbox and main bearing) and we optimise the schedule for 1 gearbox replacement and 4 main bearings replacements
  - Column 2: index of the component category: eg. O for gearbox, 1 for main bearing
  - Column 3: (optionnal, only if results on all scenario are written): index of the scenario
  - Column 4: index of the maintenance operation for the given component category (1 mean it is the second replacement in the year)

	Case	Category	Scenario	Operation	Block	StartDate
0	(4)	0	0	0	0	106
1	(4)	0	0	0	1	107
2	(4)	0	0	0	2	111
3	(4)	0	0	0	3	115
4	(4)	0	0	1	0	124
5	(4)	0	0	1	1	125
6	(4)	0	0	1	2	127
7	(4)	0	0	1	3	131
8	(4)	0	0	2	0	143
9	(4)	0	0	2	1	144
10	(4)	0	0	2	2	146
11	(4)	0	0	2	3	150
12	(4)	0	0	3	0	168
13	(4)	0	0	3	1	169
14	(4)	0	0	3	2	171
15	(4)	0	0	3	3	175

Figure 6.2: Enter Caption

  - Column 5: index of the sub-block of the current maintenance operation
  - Column 6: index of the day when the current sub-block starts

  An example is shown in figure 6.2, where the short-term model was ran for one component category and was asked to optimise 4 replacements in the year. Only one scenario (scenario 0) was included.

- Optimal cost: one csv file per year with one row per scenario and one column per number of components to replace for each component category. An example is given in figure 6.3 with 2 scenarios (scenarios 0 and 1), 2 categories of components (eg gearbox and main bearing) with different cases where 0 or 1 gearboxes and 0 to 3 main bearings are replaced.

	(0,1)	(0,2)	(0,3)	(1,0)	(1,1)	(1,2)	(1,3)
0	345	678	876	234	645	923	1234
1	341	678	823	210	602	901	1123

Figure 6.3: Enter Caption

6.4.2 Outputs of the long term model

The long term model can be ran either in deterministic mode (in this case each scenario will be optimised sequencially) or in stochastic mode (all scenarios will be optimised together). In both cases it produces both per scenario results and averaged results:

- Optimal average cost: one single value (€or £)
- Optimal cost per scenario: one csv file with one column per scenario and one row containing the optimal cost
- Optimal average schedule: one csv file per component with one row per year giving the average number of components to replace. Note that this number, as averaged, may be non integer.

- Optimal schedule per scenario: one csv file per component with X header rows (multiindex) giving the indexes of each kind of scenario, and one row per scenario giving the number of components to replace.

7 Conclusion and perspectives

This implementation will be used for the benefits of T6.2 and T6.3. In the future the following tasks could be envisaged:

- Re-write a more 'industrial' version (the one we have is a research code, usable only for the benefit of the research conducted within this project)
- Optimize the solving process, in particular for the short-term problem. For MINLP problems, the computation time can be drastically reduced by adding additionnal constraints (called cuts), which do not change the result of the optimisation, but may drastically (or not) reduce the computation time as they will help the branc and bound solver to find the optimal solution much faster. This would require consequent research time and is not feasible in the timeframe and resource allowance of the hyperwinf project but could be envisaged in the future.
- Parallelize the resolution of the short term problems. This should be pretty easy as it could be done through solving in parallel the problem on different scenarios. Nevertheless, it requires some expertise on parallelization techniques, which are out of the scope of the hyperwind project.

Acknowledgement

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